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Softly Broken Supersymmetric Desert from Orbifold Compactification

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Abstract

A new viewpoint for the gauge hierarchy problem is proposed: compactification at a large scale, $1/R$, leads to a low energy effective theory with supersymmetry softly broken at a much lower scale, α/R. The hierarchy is induced by an extremely small angle α which appears in the orbifold compactification boundary conditions. The same orbifold boundary conditions break Peccei-Quinn symmetry, leading to a new solution to the μ problem. Explicit 5d theories are constructed with gauge groups $SU(3) \times SU(2) \times U(1)$ and $SU(5)$, with matter in the bulk or on the brane, which lead to the (next-to) minimal supersymmetric standard model below the compactification scale. In all cases the soft supersymmetry-breaking and μ parameters originate from bulk kinetic energy terms, and are highly constrained. The supersymmetric flavor and CP problems are solved.

1 Introduction

Data from precision electroweak experiments, which includes evidence in favor of a light Higgs boson, have made weak scale supersymmetry the leading candidate for a theory beyond the standard model. Weak scale supersymmetry provides a solution to the gauge hierarchy problem, a radiative electroweak symmetry breaking mechanism with a light Higgs boson, and a successful prediction for the weak mixing angle. The critical question for weak scale supersymmetry is: what breaks supersymmetry? In many schemes this is accomplished in 4d by the dynamics of some new strong gauge force. In this paper we explore an alternative possibility: the breaking of supersymmetry by boundary conditions in compact extra dimensions [1]. While such a mechanism has been known for many years, it has rarely been applied to realistic models. Models which have been constructed [2, 3, 4, 5, 6, 7, 8, 9], have taken the view that the compactification scale $1/R$ is of order a TeV, and that beneath this scale supersymmetry is broken. Thus the picture is of a transition at $1/R$ from a $d > 4$ supersymmetric theory directly to a $d = 4$ non-supersymmetric effective theory. There is never an energy interval in which there is an effective 4d supersymmetric field theory. Such schemes are extremely exciting, as they predict that both Kaluza-Klein (KK) modes and superpartners will be discovered by colliders at the TeV scale. However, in these schemes supersymmetry is apparently not related to the gauge hierarchy problem, and logarithmic gauge coupling unification is not possible.

In this paper we demonstrate that there is an alternative implementation of boundary condition supersymmetry breaking: the boundary conditions may involve very small dimensionless parameters, α, so that supersymmetry is broken at α/R rather than $1/R$. In this scheme the transition at scale $1/R$ is from a 5d supersymmetric theory to a 4d supersymmetric theory with highly suppressed supersymmetry breaking interactions. In the energy interval from $1/R$ to α/R physics is described by a softly broken 4d supersymmetric theory, such as the minimal supersymmetric standard model. This new viewpoint gives a new origin for the soft supersymmetry-breaking parameters in terms of orbifold compactification boundary conditions at very high energies. For $1/R$ sufficiently high, supersymmetry is relevant for solving the gauge hierarchy problem and logarithmic gauge coupling unification may occur. We do not claim this as a new solution to the gauge hierarchy problem, as we have not understood why the parameter α is so small, but we are hopeful that this new view of the problem may lead to a new solution.

In this paper we restrict our analysis to the simplest case of a single compact extra dimension, in which case there is a unique parameter, α, in the orbifold boundary condition which breaks supersymmetry [10]. This parameter arises as a twisting of the fields under a translation symmetry of the extra coordinate. In higher dimensions there will be further parameters. In 5d, with a single extra dimension, even with arbitrary gauge and matter content of the theory, we are guaranteed that the soft supersymmetry-breaking parameters depend on only two parameters: α and $1/R$.

Realistic theories with supersymmetry in 4d must have two Higgs chiral multiplets, vectorlike with respect to the standard model gauge group. Any such theory must address why these Higgs doublets have survived to the low energy effective theory — why did they not get a large gauge invariant mass? An obvious answer is that they are protected by a global symmetry G_H — either Peccei-Quinn symmetry or R symmetry. However, in this case one must address why the level of G_H breaking scale is comparable to that of supersymmetry breaking, as required by phenomenology. We will solve these problems as follows. The Higgs fields propagate in the bulk and are forbidden to have a bulk mass term by orbifold symmetry and G_H. This same orbifold symmetry is such that, on making a KK expansion, there are two zero-mode Higgs doublets. The breaking of G_H is accomplished by an orbifold boundary condition involving a dimensionless twisting parameter γ. The relevant orbifold symmetry is precisely the same translation of the extra coordinate that breaks supersymmetry and hence one naturally expects $\gamma \approx \alpha$. There is a unification of the origin of supersymmetry and G_H breaking, providing a novel solution to the μ problem.

In the next section we study the case that the gauge group is $SU(3) \times SU(2) \times U(1)$, so that the orbifold breaks supersymmetry and global symmetry, but not gauge symmetry. We obtain the form of the supersymmetry and G_H breaking interactions, and study radiative electroweak symmetry breaking, both for the case of quarks and leptons on a brane and in the bulk.

In section 3 we study the case that the gauge group is $SU(5)$, and that the $SU(5)$ gauge symmetry is broken to that of the standard model by the same orbifold translation symmetry that breaks both supersymmetry and G_H. The gauge symmetry breaking is induced by a set of parities and does not involve any small parameter, and hence occurs at the scale of $1/R$. In the limit $\alpha, \gamma \to 0$ this theory is the same as that studied in Refs. [11, 12]. Including non-zero values for α, γ allows a unified view of gauge, global and supersymmetry breaking.

2 SU(3)×SU(2)×U(1) Model

In this section, we construct 5d theories which, at low energies, reduce to the minimal supersymmetric standard model with specific forms of the soft supersymmetry-breaking parameters.

2.1 The model

The gauge group is taken to be $SU(3) \times SU(2) \times U(1)$. The 5d gauge multiplet $\mathcal{V} = (A^M, \lambda, \lambda', \sigma)$ is decomposed into a vector superfield $V = (A^\mu, \lambda)$ and a chiral superfield $\Sigma = (\sigma + iA^5, \lambda')$ under 4d $N = 1$ supersymmetry. We also introduce two Higgs hypermultiplets $\mathcal{H}_i = (h_i, h_i^{c\dagger}, \tilde{h}_i, \tilde{h}_i^{c\dagger})$ ($i = 1, 2$) in the 5d bulk. Under 4d $N = 1$ supersymmetry, each of them is decomposed into two chiral superfields $H_i = (h_i, \tilde{h}_i)$ and $H_i^c = (h_i^c, \tilde{h}_i^c)$, where H_i and H_i^c have conjugated transformations

under the gauge group. A large bulk mass term is forbidden by imposing a global symmetry G_H under which $\tilde{h}_1$ and $\tilde{h}_2^c$ transform in the same way.

The fifth dimension is compactified on the S^1/Z_2 orbifold, which is constructed by two identifications $y \leftrightarrow -y$ and $y \leftrightarrow y+2\pi R$. Under the first identification, $y \leftrightarrow -y$, the gauge and Higgs fields are assumed to obey the following boundary conditions:

$$\begin{aligned} \begin{pmatrix} V \\ \Sigma \end{pmatrix}(x^\mu, -y) &= \begin{pmatrix} V \\ -\Sigma \end{pmatrix}(x^\mu, y), && (1) \\ \begin{pmatrix} H_1 & H_2 \\ H_1^{c\dagger} & H_2^{c\dagger} \end{pmatrix}(x^\mu, -y) &= \begin{pmatrix} H_1 & -H_2 \\ -H_1^{c\dagger} & H_2^{c\dagger} \end{pmatrix}(x^\mu, y). && (2) \end{aligned}$$

This leaves only 4d $N=1$ $SU(3) \times SU(2) \times U(1)$ vector superfields and two Higgs chiral superfields H_1 and H_2^c as zero-modes, upon compactifying to S^1/Z_2. All the other states have masses of order $1/R$. We also impose the following boundary conditions under $y \leftrightarrow y+2\pi R$:

$$\begin{aligned} A^M(x^\mu, y+2\pi R) &= A^M(x^\mu, y), && (3) \\ \begin{pmatrix} \lambda \\ \lambda' \end{pmatrix}(x^\mu, y+2\pi R) &= e^{-2\pi i\alpha\sigma_2}\begin{pmatrix} \lambda \\ \lambda' \end{pmatrix}(x^\mu, y), && (4) \\ \sigma(x^\mu, y+2\pi R) &= \sigma(x^\mu, y), && (5) \end{aligned}$$

$$\begin{aligned} \begin{pmatrix} h_1 & h_2 \\ h_1^{c\dagger} & h_2^{c\dagger} \end{pmatrix}(x^\mu, y+2\pi R) &= e^{-2\pi i\alpha\sigma_2}\begin{pmatrix} h_1 & h_2 \\ h_1^{c\dagger} & h_2^{c\dagger} \end{pmatrix} e^{2\pi i\gamma\sigma_2}(x^\mu, y), && (6) \\ \begin{pmatrix} \tilde{h}_1 & \tilde{h}_2 \\ \tilde{h}_1^{c\dagger} & \tilde{h}_2^{c\dagger} \end{pmatrix}(x^\mu, y+2\pi R) &= \begin{pmatrix} \tilde{h}_1 & \tilde{h}_2 \\ \tilde{h}_1^{c\dagger} & \tilde{h}_2^{c\dagger} \end{pmatrix} e^{2\pi i\gamma\sigma_2}(x^\mu, y). && (7) \end{aligned}$$

where α and γ are continuous parameters, and $\sigma_{1,2,3}$ are the Pauli spin matrices. Note that this is the most general boundary condition under the S^1/Z_2 compactification with the present matter content [10]; α and γ parameterize $U(1)$ rotations which are subgroups of the $SU(2)_R$ symmetry and flavor $SU(2)_H$ symmetry of the 5d action, respectively. The boundary conditions Eqs. (3 – 7) provide G_H breaking, and induce the soft supersymmetry-breaking masses of $O(\alpha/R)$ and the supersymmetric mass for the Higgs fields of $O(\gamma/R)$ as we will see below.

This theory was first introduced with the viewpoint that α and γ are of order unity, so that the theory below the compactification scale is the standard model rather than the supersymmetric standard model [4]. This led to an emphasis of the phenomenology of the case $\gamma=\alpha$, since only in this limit was a light Higgs doublet obtained [6]. Here we stress that we are interested in the very different viewpoint of α and γ being extremely small, so that there is a large energy interval in which the theory is the minimal supersymmetric standard model.

We now consider the mode expansions for the various fields under the boundary conditions Eqs. (3 – 7). Non-trivial decompositions are required for the gauginos, Higgs bosons and Higgsi-

nos. They are given by

$$\begin{pmatrix} \lambda \\ \lambda' \end{pmatrix} (x^\mu, y) = \sum_{n=0}^{\infty} e^{-i\alpha\sigma_2 y/R} \begin{pmatrix} \lambda_n \cos[ny/R] \\ \lambda'_n \sin[ny/R] \end{pmatrix}, \tag{8}$$

$$\begin{pmatrix} h_1 & h_2 \\ h_1^{c\dagger} & h_2^{c\dagger} \end{pmatrix} (x^\mu, y) = \sum_{n=0}^{\infty} e^{-i\alpha\sigma_2 y/R} \begin{pmatrix} h_{1n} \cos[ny/R] & h_{2n} \sin[ny/R] \\ h_{1n}^{c\dagger} \sin[ny/R] & h_{2n}^{c\dagger} \cos[ny/R] \end{pmatrix} e^{i\gamma\sigma_2 y/R}, \tag{9}$$

$$\begin{pmatrix} \tilde{h}_1 & \tilde{h}_2 \\ \tilde{h}_1^{c\dagger} & \tilde{h}_2^{c\dagger} \end{pmatrix} (x^\mu, y) = \sum_{n=0}^{\infty} \begin{pmatrix} \tilde{h}_{1n} \cos[ny/R] & \tilde{h}_{2n} \sin[ny/R] \\ \tilde{h}_{1n}^{c\dagger} \sin[ny/R] & \tilde{h}_{2n}^{c\dagger} \cos[ny/R] \end{pmatrix} e^{i\gamma\sigma_2 y/R}, \tag{10}$$

where $\lambda_n, \lambda'_n, h_{in}, h_{in}^c, \tilde{h}_{in}$ and $\tilde{h}_{in}^c$ are 4d fields. On substituting these mode expansions into the 5d action and integrating out the heavy modes with masses of $O(1/R)$, we obtain the 4d effective Lagrangian below the scale of $1/R$. It contains only the $SU(3) \times SU(2) \times U(1)$ vector superfields and two Higgs chiral superfields H_1 and H_2^c, which we define as $H_u \equiv H_{1,n=0}$ and $H_d \equiv H_{2,n=0}^c$. In addition to the kinetic terms for these fields, there are mass terms coming from the boundary conditions Eqs. (3 – 7),

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}\frac{\alpha}{R}(\lambda_0^a \lambda_0^a + \text{h.c.}) \\ &\quad -\left(\frac{\alpha^2}{R^2} + \frac{\gamma^2}{R^2}\right)(|h_u|^2 + |h_d|^2) + 2\frac{\alpha\gamma}{R^2}(h_u h_d + \text{h.c.}) \\ &\quad -\frac{\gamma}{R}(\tilde{h}_u \tilde{h}_d + \text{h.c.}), \end{aligned} \tag{11}$$

where various fields are canonically normalized in 4d, and a runs over $SU(3)$, $SU(2)$ and $U(1)$. We find that the gaugino masses, soft supersymmetry-breaking masses for the Higgs bosons, the supersymmetric Higgs mass (μ term) and the holomorphic supersymmetry-breaking Higgs mass (μB term) are generated. Therefore, the low energy theory has the structure of the minimal supersymmetric standard model with various relations on the soft supersymmetry-breaking parameters. In particular, it determines the sign of the μ parameter in the basis where $\langle h_u \rangle, \langle h_d \rangle > 0$. The interactions in Eq. (11) are such that, using conventional notation, the sign of μ is negative. (In the conventional notation, a negative μ leads to a stronger constraint from $b \to s\gamma$.) An interesting point is that the sizes of α and γ are expected to be the same order, since the $U(1)$ symmetry used to twist the boundary condition under $y \leftrightarrow y + 2\pi R$ is a generic linear combination of two $U(1)$ symmetries, $U(1)_R \subset SU(2)_R$ and $U(1)_H \subset SU(2)_H$, associated with α and γ. Therefore, this theory provides a natural solution to the μ problem.

Orbifold breaking has led to a soft rather than hard breaking of supersymmetry. When the KK mode expansions of Eqs. (8, 9) are substituted into the kinetic energy, the y derivatives give α/R and result in soft operators, while the 4d derivatives do not lead to supersymmetry breaking.[1]

[1] We thank Alex Pomarol for pointing out our previous error on this point.

The supersymmetry-breaking parameter α/R drops out of the 4d kinetic terms (kinetic terms with ∂_μ) because of $SU(2)_R$ invariance. It does not drop out of the bulk kinetic term (kinetic terms with ∂_y) because $SU(2)_R$ is a global symmetry, and the phase factors in Eqs. (8, 9) are y dependent. The resulting supersymmetry breaking interactions are soft by dimensional analysis: the derivative ∂_y becomes the soft supersymmetry-breaking parameter. Hard supersymmetry breaking effects do not arise from the minimal kinetic terms in the 5d bulk.

So far, we have considered the gauge and Higgs fields. How are quarks and leptons incorporated into the above theory? There are essentially two ways to introduce quarks and leptons into the model: as fields on the brane or in the bulk. Here we concentrate on the case that the quarks and leptons are placed on a fixed point of the S^1/Z_2 orbifold, which, without loss of generality, we take to be at $y = 0$. The case of quarks and leptons in the bulk are considered in sub-section 2.3. Then, quark and lepton chiral superfields, Q, U, D, L and E, are introduced on the $y = 0$ brane, together with appropriate Yukawa couplings with the Higgs fields in the bulk,

$$S = \int d^4x\; dy\; \delta(y) \left[\int d^2\theta \, (y_u QUH_1 + y_d QDH_2^c + y_e LEH_2^c) + \text{h.c.} \right]. \qquad (12)$$

With these Yukawa couplings, the theory precisely reduces to the minimal supersymmetric standard model at low energies. Note that since the squarks and sleptons are brane fields, their masses are not generated by the orbifolding; soft supersymmetry-breaking masses for squarks and sleptons are essentially zero at the scale of $1/R$. However, they are radiatively generated through renormalization group equations below the scale of $1/R$. Since the radiative corrections are almost flavor universal, the supersymmetric flavor problem is solved in this model.

To summarize, the present model gives the minimal supersymmetric standard model at low energies, with a constrained form of soft supersymmetry-breaking parameters. They are given, at the scale $1/R$, by

$$m_{1/2} = \hat{\alpha} \equiv \alpha/R, \qquad (13)$$

$$m^2_{h_u,h_d} = \hat{\alpha}^2, \qquad m^2_{\tilde{q},\tilde{u},\tilde{d},\tilde{l},\tilde{e}} = 0, \qquad A = -\hat{\alpha}, \qquad (14)$$

$$\mu = \hat{\gamma} \equiv \gamma/R, \qquad \mu B = -2\hat{\alpha}\hat{\gamma}. \qquad (15)$$

where $m_{1/2}$ represents the universal gaugino mass and A the trilinear scalar couplings. The predicted sign of A is such that, on scaling to the infrared, $|A|$ is increased by the radiative contribution from the gaugino mass. Here, we have neglected threshold effects coming from finite radiative corrections at $1/R$. In this expression, while the compactification radius R is an arbitrary parameter, $\hat{\alpha}$ and $\hat{\gamma}$ must be around the weak scale for the supersymmetry to be relevant as a solution to the gauge hierarchy problem. One interesting consequence of Eq. (13) is that the gaugino masses are unified at the scale $1/R$. This arises because in 5d the most general orbifolding admits only a single parameter which breaks supersymmetry. In general the compactification scale differs from the unification scale, so that the gaugino masses do not unify

at the grand unification scale. By construction, all the above quantities are necessarily real, so that there is no supersymmetric CP problem. Below, we consider all the range of $1/R$ from the weak to the Planck scale, treating $\hat{\alpha}$ and $\hat{\gamma}$ as free parameters of the order of the weak scale.

2.2 Radiative electroweak symmetry breaking

Having obtained soft supersymmetry-breaking parameters, Eqs. (13 – 15), at the compactification scale, we can solve renormalization group equations to obtain the spectrum at the weak scale. In particular, we can work out whether radiative electroweak symmetry breaking occurs correctly or not. In this sub-section, we will consider the constraint from radiative electroweak symmetry breaking and find that it gives a restriction on the values for $1/R$, $\hat{\alpha}$ and $\hat{\gamma}$.

The minimization of the Higgs potential gives two relations,

$$\frac{m_Z^2}{2} = \frac{\tan^2\beta\, m_{h_u}^2 - m_{h_d}^2}{1-\tan^2\beta} - |\mu|^2, \tag{16}$$

$$\sin(2\beta) = -\frac{2\mu B}{m_{h_u}^2 + m_{h_d}^2 + 2|\mu|^2}, \tag{17}$$

where various quantities must be evaluated at the weak scale. Using these equations, we can relate two parameters $\hat{\alpha}$ and $\hat{\gamma}$ with $\tan\beta \equiv \langle h_u \rangle / \langle h_d \rangle$ and $v \equiv \sqrt{\langle h_u \rangle^2 + \langle h_d \rangle^2}$. Since v is fixed by the observed Fermi constant, there are two remaining free parameters which we take to be $1/R$ and $\tan\beta$. Below, we consider the constraint on this two dimensional parameter space from electroweak symmetry breaking. We look for solutions of Eqs. (16, 17) with $\tan\beta \gtrsim 2$, to satisfy the experimental lower bound on the physical Higgs boson mass.

A characteristic feature of the soft supersymmetry-breaking parameters given in Eqs. (13 – 15) is a sizable non-vanishing value for the μB parameter. The μB parameter pushes the value of $\tan\beta$ towards 1. Thus we want to reduce the effect of the μB parameter relative to that of the other parameters, which requires a hierarchy between the two parameters $\hat{\alpha}$ and $\hat{\gamma}$. This can be seen easily as follows. Suppose, as a zero-th order approximation, that only $m_{h_u}^2$ is changed through renormalization group evolution from $1/R$ to the weak scale. Then, relevant parameters are given by $m_{h_u}^2 = (1-c)\hat{\alpha}^2$, $m_{h_d}^2 = \hat{\alpha}^2$, $\mu = \hat{\gamma}$ and $\mu B = -2\hat{\alpha}\hat{\gamma}$ at the weak scale. Here, c parameterizes the renormalization scaling induced by the top Yukawa coupling, and depends on $1/R$ and $\tan\beta$ through the distance of renormalization group running and the size of the top Yukawa coupling, respectively. After this scaling, Eq. (17) becomes

$$\frac{\tan\beta}{1+\tan^2\beta} \simeq \frac{\hat{\alpha}\hat{\gamma}}{(1-c/2)\hat{\alpha}^2 + \hat{\gamma}^2}, \tag{18}$$

and $\tan\beta \gg 1$ requires either $\hat{\alpha}/\hat{\gamma} \ll 1$ or $\hat{\gamma}/\hat{\alpha} \ll 1$. Although the above argument is very rough, numerical computations confirm that successful electroweak symmetry breaking with $\tan\beta \gtrsim 2$ requires a hierarchy between $\hat{\alpha}$ and $\hat{\gamma}$ of typically an order of magnitude.

We first consider the case $\hat{\alpha}/\hat{\gamma} \ll 1$. In this case, electroweak symmetry breaking does not occur, since the supersymmetric mass term for the Higgs fields is much larger than the supersymmetry-breaking masses which would trigger the electroweak symmetry breaking. In other words, the right-hand side of Eq. (16) formally gives a negative value and is unphysical. Thus we concentrate on the case $\hat{\gamma}/\hat{\alpha} \ll 1$ from now on. With $\hat{\gamma}/\hat{\alpha} \ll 1$, the values for $\hat{\alpha}$ and $\hat{\gamma}$ are given by $\hat{\gamma} \ll \hat{\alpha} \sim m_Z$ in a generic region of the parameter space, so that it is not phenomenologically acceptable. This can be easily seen by noting that Eq. (16) reduces, with moderately large values for $\tan\beta$, to

$$\frac{m_Z^2}{2} \simeq -(1-c)\hat{\alpha}^2 - \hat{\gamma}^2, \tag{19}$$

in the approximation adopted in Eq. (18). However, Eq. (19) also provides the way to avoid this problem. If $(c-1)\hat{\alpha}^2 \simeq \hat{\gamma}^2$, which means $c \simeq 1$, we can obtain $\hat{\gamma} \sim m_Z$ or even larger values for $\hat{\gamma}$. Then, since c depends on both $1/R$ and $\tan\beta$, $c \simeq 1$ gives one constraint on these values: a phenomenologically acceptable parameter region is a line in the two dimensional parameter space spanned by $1/R$ and $\tan\beta$.

The actual dependence of c on $1/R$ and $\tan\beta$ is somewhat complicated, and also the approximation in Eq. (18) is not very precise. Thus we have evaluated the allowed region by numerical computations, including full renormalization group effects at the two-loop level. We find that the allowed region is a curved line, which extends from $(1/R, \tan\beta) \sim (2 \times 10^6~\text{GeV}, 2)$ through $(1/R, \tan\beta) \sim (3 \times 10^7~\text{GeV}, 5)$ to $(1/R, \tan\beta) \sim (6 \times 10^7~\text{GeV}, 20)$. The thickness of this line is given by $\delta(\tan\beta)/\tan\beta \sim 5\%$, with a fixed value of $1/R$. This behavior is easily understood in terms of the correlation between $1/R$ and $\tan\beta$ required to maintain c close to unity. If $1/R$ is below 2×10^6 GeV, the running distance is short so that $c \simeq 1$ requires fairly large values for the top Yukawa coupling, corresponding to $\tan\beta \lesssim 2$. Thus there is no phenomenologically acceptable parameter region for $1/R \lesssim 2 \times 10^6$ GeV. Once $1/R$ is increased above 2×10^6 GeV, an allowed parameter region emerges, giving correct radiative electroweak symmetry breaking with $\tan\beta \gtrsim 2$. As $1/R$ is further increased, the running distance also increases, and thus $c \simeq 1$ requires smaller values of the top Yukawa coupling, corresponding to larger $\tan\beta$. Thus the allowed parameter region extends to the upper right direction in the $(1/R, \tan\beta)$ plane. However, the top Yukawa coupling cannot be made arbitrary small, since its dependence on $\tan\beta$ is very weak for $\tan\beta \gtrsim 20$, leading to an upper bound: $1/R \lesssim 6 \times 10^7$ GeV.

In the region discussed above, the low energy B parameter is large, reflecting the large value at the compactification scale. However, the sign of A is such that the magnitude of B is reduced during evolution to the infrared; hence for large enough $1/R$ we find an acceptable region with a low value for B at the weak scale. This requires $1/R \gtrsim 10^{14}$ GeV, in which case we find successful electroweak symmetry breaking occurs only for $\tan\beta$ near 2. Since $\hat{\gamma}/\hat{\alpha}$ is now of order unity, there is no need for $c \simeq 1$. However, for $\tan\beta \simeq 2$, a sufficiently heavy Higgs boson only results

for heavy squarks, so $\hat{\alpha}$ must be large. Hence some cancellation between the $\hat{\alpha}^2$ and $\hat{\gamma}^2$ terms are required in Eq. (19).

In summary, we have found two regions of parameter space where correct electroweak symmetry breaking occurs in the present model. In one region, the compactification scale is tightly constrained, 2×10^6 GeV $\lesssim 1/R \lesssim 6 \times 10^7$ GeV, while a broad region of $\tan\beta$, $2 \lesssim \tan\beta \lesssim 20$, can be realized depending on the value of $1/R$. This somewhat unusual result is a consequence of the sizable μB parameter at the compactification scale. In the other region, $1/R \gtrsim 10^{14}$ GeV and $\tan\beta \simeq 2$.

The above constraint on $1/R$ and $\tan\beta$ is derived by considering only the usual logarithmic renormalization group evolutions. There are also finite one-loop radiative corrections to the soft supersymmetry-breaking parameters that do not involve a log factor. In a 5d calculation, these appear as threshold effects at $1/R$. In the 4d picture this corresponds to including supersymmetry breaking effects from higher KK modes. These contributions are expected to be of $O(\hat{\alpha}^2/16\pi^2)$ and thus smaller than those calculated above by an amount of order $1/\ln(1/\alpha)$. To find whether they are negligible or significant, however, a full one-loop calculation must be done to include the effects of the heavier modes of the KK tower so that a finite answer is obtained. In the case that the usual logarithmic term dominates, the allowed range of $1/R$ quoted above will be unchanged.

Finally, we comment on the effect of brane-localized kinetic terms for the Higgs fields,

$$S = \int d^4x\, dy\, \delta(y) \int d^4\theta \left(Z_u H_u^\dagger H_u + Z_d H_d^\dagger H_d \right). \tag{20}$$

This changes the Higgs potential and Higgsino mass given in Eq. (11) to

$$\begin{aligned} \mathcal{L} \;=\; & -\left(\frac{\alpha_u'^2}{R^2} + \frac{\gamma'^2}{R^2} \right) |h_u|^2 - \left(\frac{\alpha_d'^2}{R^2} + \frac{\gamma'^2}{R^2} \right) |h_d|^2 \\ & + \frac{(\alpha_u' + \alpha_d')\gamma'}{R^2} (h_u h_d + \text{h.c.}) - \frac{\gamma'}{R} (\tilde{h}_u \tilde{h}_d + \text{h.c.}). \end{aligned} \tag{21}$$

Here, α_u', α_d' and γ' are given by

$$\alpha_u' = \frac{\alpha}{1+z_u}, \qquad \alpha_d' = \frac{\alpha}{1+z_d}, \qquad \gamma' = \frac{\gamma}{\sqrt{1+z_u}\sqrt{1+z_d}}, \tag{22}$$

where $z_u \equiv Z_u/2\pi R$ and $z_d \equiv Z_d/2\pi R$. However, we expect that z_u and z_d are small, since they are suppressed by the length of the extra dimension. We have checked that the inclusion of the brane kinetic terms does not change the qualitative feature of the analysis in this sub-section unless they are large, $z_u, z_d \gtrsim O(1)$.

2.3 Quarks and leptons in the bulk

In this sub-section, we consider the case where the quarks and leptons are put in the bulk, rather than on the $y = 0$ brane. In this case, we have to introduce hypermultiplets $\mathcal{Q}_j, \mathcal{U}_j, \mathcal{D}_j, \mathcal{L}_j$ and $\mathcal{E}_j$

in the 5d bulk, where $j = 1, 2, 3$ represents the generation index. Each of them is decomposed, under 4d $N = 1$ supersymmetry, into two chiral superfields as $(Q_j, Q_j^c), (U_j, U_j^c), (D_j, D_j^c), (L_j, L_j^c)$ and (E_j, E_j^c) where conjugated fields have conjugate transformations under the gauge group.

The boundary conditions under the orbifolding are given as follows. We must require that the orbifolding yields three light generations of chiral matter. This uniquely determines that, under $y \leftrightarrow -y$, $\mathcal{Q}_j$'s obey

$$\begin{pmatrix} Q_1 & Q_2 & Q_3 \\ Q_1^{c\dagger} & Q_2^{c\dagger} & Q_3^{c\dagger} \end{pmatrix}(x^\mu, -y) = \begin{pmatrix} Q_1 & Q_2 & Q_3 \\ -Q_1^{c\dagger} & -Q_2^{c\dagger} & -Q_3^{c\dagger} \end{pmatrix}(x^\mu, y). \tag{23}$$

and also that, under $y \leftrightarrow y + 2\pi R$

$$\begin{pmatrix} \tilde{q}_1 & \tilde{q}_2 & \tilde{q}_3 \\ \tilde{q}_1^{c\dagger} & \tilde{q}_2^{c\dagger} & \tilde{q}_3^{c\dagger} \end{pmatrix}(x^\mu, y + 2\pi R) = e^{-2\pi i\alpha\sigma_2}\begin{pmatrix} \tilde{q}_1 & \tilde{q}_2 & \tilde{q}_3 \\ \tilde{q}_1^{c\dagger} & \tilde{q}_2^{c\dagger} & \tilde{q}_3^{c\dagger} \end{pmatrix}(x^\mu, y), \tag{24}$$

$$\begin{pmatrix} q_1 & q_2 & q_3 \\ q_1^{c\dagger} & q_2^{c\dagger} & q_3^{c\dagger} \end{pmatrix}(x^\mu, y + 2\pi R) = \begin{pmatrix} q_1 & q_2 & q_3 \\ q_1^{c\dagger} & q_2^{c\dagger} & q_3^{c\dagger} \end{pmatrix}(x^\mu, y), \tag{25}$$

where $Q_j = (\tilde{q}_j, q_j)$ and $Q_j^c = (\tilde{q}_j^c, q_j^c)$. The uniqueness of this choice is non-trivial. A twisting of the fields in flavor space under $y \leftrightarrow y + 2\pi R$ is only consistent if the fields have opposite parities under $y \leftrightarrow -y$ [10]. However, having opposite parities yields vector-like matter at low energy, as can be seen from the Higgs case. The other fields, $\mathcal{U}_j, \mathcal{D}_j, \mathcal{L}_j$ and $\mathcal{E}_j$, must obey the same boundary conditions. With these boundary conditions, the matter content below $1/R$ scale is exactly the three generations of quark and lepton chiral superfields.

The mode expansions for the squarks are given by

$$\begin{pmatrix} \tilde{q}_j \\ \tilde{q}_j^{c\dagger} \end{pmatrix}(x^\mu, y) = \sum_{n=0}^{\infty} e^{-i\alpha\sigma_2 y/R}\begin{pmatrix} \tilde{q}_{jn}\cos[ny/R] \\ \tilde{q}_{jn}^{c\dagger}\sin[ny/R] \end{pmatrix}. \tag{26}$$

The expansions for the quarks are straightforward (corresponding to $\alpha = 0$ in the above equation). Identifying $n = 0$ modes with the usual squarks (and sleptons), we obtain the following soft supersymmetry-breaking masses at the $1/R$ scale:

$$\mathcal{L} = -\frac{\alpha^2}{R^2}\sum_{j=1}^{3}\left(|\tilde{q}_j|^2 + |\tilde{u}_j|^2 + |\tilde{d}_j|^2 + |\tilde{l}_j|^2 + |\tilde{e}_j|^2\right), \tag{27}$$

where squark and slepton fields are canonically normalized in 4d. We find that the universal scalar mass, α/R, is generated. This degeneracy among various squark and slepton masses is lifted by the presence of the brane-localized kinetic terms. However, we expect that these terms are small due to the volume suppression from the extra dimension, so that these theories offer an interesting way to solve the supersymmetric flavor problem.

As in the case of brane matter, the Yukawa couplings Eq. (12) are introduced on the $y = 0$ brane. Then, below the compactification scale $1/R$, the theory reduces to the minimal supersymmetric standard model with

$$m_{1/2} = \hat{\alpha}, \qquad m^2_{h_u,h_d} = m^2_{\tilde{q},\tilde{u},\tilde{d},\tilde{l},\tilde{e}} = \hat{\alpha}^2, \qquad A = -3\hat{\alpha}, \tag{28}$$

$$\mu = \hat{\gamma}, \qquad \mu B = -2\hat{\alpha}\hat{\gamma}, \tag{29}$$

Again, there are neither grand unified relations among the gaugino masses (unless $1/R$ is close to the unification scale) nor supersymmetric flavor or CP problems.

In contrast to the case of brane matter, squarks and sleptons have non-vanishing soft masses at the compactification scale and the A parameter is very large. This substantially changes the numerical results for successful electroweak symmetry breaking, although the qualitative behavior is the same: the allowed parameter region is a line in the $(1/R, \tan\beta)$ plane with the two quantities positively correlated. There are both a low $1/R$ region, having c close to unity and large B, and a high $1/R$ region, with c not near unity and smaller values for B. In the low $1/R$ region, the values for $1/R$ are much lower than those in the brane matter case, since the non-vanishing top squark masses at the compactification scale and the large A parameter lead to large radiative corrections for the Higgs mass, so that $c \simeq 1$ is obtained with a shorter running distance. Successful electroweak symmetry breaking occurs in this region with $700~\text{GeV} \lesssim 1/R \lesssim 2~\text{TeV}$, with $2 \lesssim \tan\beta \lesssim 20$. The second region is larger than before, since small B is obtained with less running due to the larger value for A. This region extends from $(1/R, \tan\beta) \sim (10^7~\text{GeV}, 2)$ to $(1/R, \tan\beta) \sim (10^{16}~\text{GeV}, 4)$. We conclude that there is now a preferred region: $1/R$ can be identified with the unification scale, and the resulting value for $\tan\beta$ is sufficiently large that the Higgs mass bound does not require the squarks to be very heavy, so that electroweak symmetry breaking occurs with little fine tuning. Finally we note that there is a small third region having small B giving $(1/R, \tan\beta) \sim (10^{16}~\text{GeV}, 30)$, so that the b-quark Yukawa coupling is sufficiently large to affect the scaling of the Higgs mass parameters.

2.4 Alternative sources for μ

So far in this paper we have assumed that both supersymmetry breaking and G_H breaking arise from boundary conditions at the scale $1/R$. Here we comment briefly on alternative sources for μ, while preserving supersymmetry breaking from the boundary condition parameter α. If μ arises from physics at shorter distances than $1/R$, then it will appear in the five dimensional theory as a brane localized operator $\delta(y)\mu H_1 H_2^c$. In this case the compactification leads to $B = -2\hat{\alpha}$ at the scale $1/R$, so that the regions for successful electroweak symmetry breaking are precisely those discussed in the previous two sub-sections. However, if μ is generated in the four dimensional effective theory below $1/R$, then other values of B will occur in general, changing the conditions for electroweak symmetry breaking.

As an example of low scale μ generation, we consider a theory which is similar to the next-to-minimal supersymmetric standard model. We introduce a singlet chiral superfield S on the $y=0$ brane and couple it with the Higgs fields as

$$S = \int d^4x \; dy \; \delta(y) \left[\int d^2\theta \left(\lambda \, S H_1 H_2^c + \frac{k}{3} S^3 \right) + \text{h.c.} \right]. \qquad (30)$$

A tree level μ parameter may be forbidden by an R or discrete Z_3 symmetry, or by the requirement that the superpotential not contain any mass parameter. At the compactification scale, $A_\lambda = -2\hat{\alpha}$ and $A_k = m_s^2 = 0$. The Higgs fields have soft supersymmetry-breaking masses at the compactification scale, so that renormalization group scaling below $1/R$ drives m_s^2 negative. The resulting vacuum expectation values for the scalar and F components of S generate effective μ and μB parameters, respectively. Since these expectation values depend on the coupling constants λ and k, the μ and μB parameters are essentially free parameters in this model. Therefore, we can evade the stringent constraints on $(1/R, \tan\beta)$ derived in the previous sub-sections. However, supersymmetry breaking is still determined by the single parameter $\hat{\alpha}$, so that the tight predictions for squark, slepton and gaugino masses still apply.

3 Embedding into SU(5)

In this section, we construct 5d $SU(5)$ theories which reduce to the softly broken minimal supersymmetric standard model at low energies. The structure of the theories is similar to the 5d $SU(5)$ model discussed in Refs. [11, 12]. However, the orbifold boundary conditions are modified using $U(1)_R$ and $U(1)_H$, giving simultaneous breakings of both supersymmetry and $SU(5)$ gauge symmetry from a single orbifolding.

3.1 The model

We consider a 5d $SU(5)$ gauge theory compactified on the S^1/Z_2 orbifold. The radius of the fifth dimension is taken to be around the grand unification scale, $1/R \sim 10^{16}$ GeV, as we will see later. We also introduce two Higgs hypermultiplets, $\mathcal{H}_{\mathbf{5_1}} = (H_{\mathbf{5_1}}, H_{\mathbf{5_1}}^{c\dagger})$ and $\mathcal{H}_{\mathbf{5_2}} = (H_{\mathbf{5_2}}, H_{\mathbf{5_2}}^{c\dagger})$, in the bulk, each transforming as a fundamental representation of $SU(5)$.

What accomplishes the $SU(5)$ breaking? We impose that the gauge and Higgs fields transform as $\mathcal{V} \rightarrow P\mathcal{V}P^{-1}$ and $\mathcal{H}_{\mathbf{5_{1,2}}} \rightarrow P\mathcal{H}_{\mathbf{5_{1,2}}}$ under $y \rightarrow y + 2\pi R$. Here, P is a diagonal matrix acting on the index of the fundamental representation, $P = \text{diag}(-,-,-,+,+)$. This gives masses of order $1/R$ for the gauge bosons of $SU(5)/(SU(3)\times SU(2)\times U(1))$ and for the triplet Higgs fields, so that the effective field theory below $1/R$ is that of a 4d, $N=1$, $SU(3)\times SU(2)\times U(1)$ gauge theory with two Higgs doublets. An important point here is that this boundary conditions are compatible with those discussed in the previous section which were used to break supersymmetry and give

the μ term. That is, we can simultaneously impose both $SU(5)$ breaking and supersymmetry breaking boundary conditions.

To show how the above construction works explicitly, let us label the gauge fields of $SU(3)\times SU(2)\times U(1)$ and $SU(5)/(SU(3)\times SU(2)\times U(1))$ as $\mathcal{V}^{(+)}=(V^{(+)},\Sigma^{(+)})$ and $\mathcal{V}^{(-)}=(V^{(-)},\Sigma^{(-)})$, respectively. Similarly, we represent the doublet and triplet components of the Higgs hypermultiplets by the superscript $(+)$ and $(-)$, respectively: $\mathcal{H}_{\mathbf{5_i}}\to\mathcal{H}_i^{(\pm)}=(H_i^{(\pm)},H_i^{(\pm)c\dagger})$ where $i=1,2$. Then, the boundary conditions are explicitly represented as follows. Under $y\leftrightarrow -y$, the fields must satisfy

$$\begin{aligned}
\begin{pmatrix} V^{(\pm)} \\ \Sigma^{(\pm)} \end{pmatrix}(x^\mu,-y) &= \begin{pmatrix} V^{(\pm)} \\ -\Sigma^{(\pm)} \end{pmatrix}(x^\mu,y), &(31)\\
\begin{pmatrix} H_1^{(\pm)} & H_2^{(\pm)} \\ H_1^{(\pm)c\dagger} & H_2^{(\pm)c\dagger} \end{pmatrix}(x^\mu,-y) &= \begin{pmatrix} H_1^{(\pm)} & -H_2^{(\pm)} \\ -H_1^{(\pm)c\dagger} & H_2^{(\pm)c\dagger} \end{pmatrix}(x^\mu,y), &(32)
\end{aligned}$$

and, under $y\leftrightarrow y+2\pi R$, they obey

$$\begin{aligned}
A^{(\pm)M}(x^\mu,y+2\pi R) &= \pm\, A^{(\pm)M}(x^\mu,y), &(33)\\
\begin{pmatrix} \lambda^{(\pm)} \\ \lambda'^{(\pm)} \end{pmatrix}(x^\mu,y+2\pi R) &= \pm\, e^{-2\pi i\alpha\sigma_2}\begin{pmatrix} \lambda^{(\pm)} \\ \lambda'^{(\pm)} \end{pmatrix}(x^\mu,y), &(34)\\
\sigma^{(\pm)}(x^\mu,y+2\pi R) &= \pm\,\sigma^{(\pm)}(x^\mu,y), &(35)
\end{aligned}$$

$$\begin{aligned}
\begin{pmatrix} h_1^{(\pm)} & h_2^{(\pm)} \\ h_1^{(\pm)c\dagger} & h_2^{(\pm)c\dagger} \end{pmatrix}(x^\mu,y+2\pi R) &= \pm\, e^{-2\pi i\alpha\sigma_2}\begin{pmatrix} h_1^{(\pm)} & h_2^{(\pm)} \\ h_1^{(\pm)c\dagger} & h_2^{(\pm)c\dagger} \end{pmatrix} e^{2\pi i\gamma\sigma_2}(x^\mu,y), &(36)\\
\begin{pmatrix} \tilde{h}_1^{(\pm)} & \tilde{h}_2^{(\pm)} \\ \tilde{h}_1^{(\pm)c\dagger} & \tilde{h}_2^{(\pm)c\dagger} \end{pmatrix}(x^\mu,y+2\pi R) &= \pm\begin{pmatrix} \tilde{h}_1^{(\pm)} & \tilde{h}_2^{(\pm)} \\ \tilde{h}_1^{(\pm)c\dagger} & \tilde{h}_2^{(\pm)c\dagger} \end{pmatrix} e^{2\pi i\gamma\sigma_2}(x^\mu,y). &(37)
\end{aligned}$$

Here, we are considering $\alpha\sim\gamma\ll 1$ such that $\alpha/R\sim\gamma/R$ are around the weak scale.

In the limit $\alpha,\gamma\to 0$, the above boundary conditions give the following mass spectrum [11]. The fields in the minimal supersymmetric standard model, $V^{(+)}$, $H_1^{(+)}$ and $H_2^{(+)c}$, have a tower with masses given by n/R $(n=0,1,2,\cdots)$; similarly, $(n+1/2)/R$ for $V^{(-)}$, $\Sigma^{(-)}$, $H_1^{(-)}$, $H_1^{(-)c}$, $H_2^{(-)}$ and $H_2^{(-)c}$, and $(n+1)/R$ for $\Sigma^{(+)}$, $H_1^{(+)c}$ and $H_2^{(+)}$. Therefore, in this limit, we have massless fields, $V^{(+)}$, $H_1^{(+)}\equiv H_u$ and $H_2^{(+)c}\equiv H_d$, which we call quasi zero-modes. Furthermore, since the broken gauge fields have masses of $O(1/R)$, the compactification scale is of order the unification scale.

When we turn on tiny non-zero values for $\alpha\sim\gamma$, they perturb the mass spectrum of the towers by an amount $O(\alpha/R\sim\gamma/R)$. In particular, it gives the soft supersymmetry-breaking masses for the quasi zero-modes and the μ term. Since the boundary conditions for the quasi zero-modes are the same as those discussed in section 2, the effective 4d Lagrangian for the supersymmetry and G_H breaking interactions below $1/R$ is given by Eq. (11). This forces us to take $\alpha/R\sim\gamma/R$ around the weak scale, that is $\alpha\sim\gamma\sim 10^{-13}$.

Before introducing quarks and leptons into the model, let us note one important difference between the $SU(3) \times SU(2) \times U(1)$ model and the present $SU(5)$ model. In the $SU(3) \times SU(2) \times U(1)$ case, the gaugino masses are unified at the compactification scale, so that there is generically no grand unified relation among them. On the other hand, in the $SU(5)$ case, the grand unified relation $m_{\lambda_{SU(3)}}/\alpha_{SU(3)} = m_{\lambda_{SU(2)}}/\alpha_{SU(2)} = m_{\lambda_{U(1)}}/\alpha_{U(1)}$ necessarily holds. This is true even in the presence of $SU(5)$-violating gauge kinetic terms that can be introduced on the $y = \pi R$ brane. The argument is the following. Suppose we have both the bulk gauge kinetic term, which must be $SU(5)$ symmetric, and the brane-localized gauge kinetic terms at $y = \pi R$, which can have different coefficients for $SU(3)$, $SU(2)$ and $U(1)$. Then, the 4d gauge couplings g_a are given by $1/g_a^2 = 2\pi R/g_5^2 + 1/g_{4,a}^2$, where g_5 and $g_{4,a}^2$ are the bulk and brane gauge couplings, respectively, and a runs over $SU(3)$, $SU(2)$ and $U(1)$. Thus these gauge couplings are not necessarily unified exactly at the compactification scale, although the $SU(5)$-violating piece is volume suppressed and small [12]. On the other hand, the gaugino masses $m_{\lambda,a}$ are given by $m_{\lambda,a}/g_a^2 = (2\pi R/g_5^2)(\alpha/R)$. This shows that the quantities $m_{\lambda,a}/g_a^2$ are universal, and thus the grand unified relation on the gaugino masses holds very precisely.

Let us now discuss the quarks and leptons. As in the case of $SU(3) \times SU(2) \times U(1)$, we can introduce them either on the brane or in the bulk. We first consider the case of brane matter, and defer the case of bulk matter to the next sub-section. To obtain the usual $SU(5)$ understanding of quark and lepton quantum numbers, we introduce matter chiral superfields $T_{\mathbf{10_j}}$ and $F_{\mathbf{\bar{5}_j}}$ on the $y = 0$ brane, where $j = 1, 2, 3$ is the generation index. Then, we can write down the $SU(5)$ symmetric Yukawa couplings on the $y = 0$ brane [11, 12][2]

$$S = \int d^4x \, dy \, \delta(y) \left[\int d^2\theta \sum_{j,k=1}^{3} \Big((y_1)_{jk} T_{\mathbf{10_j}} T_{\mathbf{10_k}} H_{\mathbf{5_1}} + (y_2)_{jk} T_{\mathbf{10_j}} F_{\mathbf{\bar{5}_k}} H^c_{\mathbf{5_2}} \Big) + \text{h.c.} \right]. \qquad (38)$$

With these Yukawa couplings, the theory reduces, below the compactification scale, to the minimal supersymmetric standard model with the soft supersymmetry-breaking parameters (and the μ parameter) given by Eqs. (13 – 15). Here electroweak symmetry breaking is as before except now we require $1/R \approx 10^{16}$ GeV so that $\tan\beta \simeq 2$. The Higgs mass bound is only satisfied for somewhat heavy squarks, giving some fine tuning in electroweak symmetry breaking.

We finally comment on the phenomenologies of the present model. First of all, the dangerous dimension 5 proton decay operators are not generated by an exchange of the triplet Higgs multiplets, due to the specific form of the triplet Higgs mass terms [12]. Furthermore, unwanted tree-level brane operators at $y = 0$, such as $[H_{\mathbf{5_1}} H^c_{\mathbf{5_2}}]_{\theta^2}$, $[T_{\mathbf{10_j}} T_{\mathbf{10_k}} T_{\mathbf{10_l}} F_{\mathbf{\bar{5}_m}}]_{\theta^2}$, $[F_{\mathbf{\bar{5}_j}} H_{\mathbf{5_1}}]_{\theta^2}$ and $[T_{\mathbf{10_j}} F_{\mathbf{\bar{5}_k}} F_{\mathbf{\bar{5}_l}}]_{\theta^2}$, are forbidden by imposing $U(1)_R$ symmetry on the theory [12]. (In the case of the non-minimal theory, with the superpotential of Eq. (30), the $U(1)_R$ symmetry given in Ref. [12] is explicitly broken to a $Z_{4,R}$ subgroup, which is still sufficient to forbid these un-

[2] These brane interactions are different from those adopted in Ref. [13].

wanted operators. An alternative way is to consider a different $\mathrm{U}(1)_R$ symmetry, under which $\{H_{\mathbf{5_1}}, H^c_{\mathbf{5_2}}, T_{\mathbf{10_j}}, F_{\bar{\mathbf{5}}_\mathbf{k}}, S\}$ and $\{H^c_{\mathbf{5_1}}, H_{\mathbf{5_2}}\}$ have charges $2/3$ and $4/3$, respectively.) This $U(1)_R$ symmetry (or $Z_{4,R}$) is weakly broken to R-parity subgroup by the orbifold boundary conditions, so that the gaugino masses and the μ parameter are generated. The value of $1/R$ is lower than the conventional grand unification scale $\simeq 2 \times 10^{16}$ GeV due to the threshold effect coming from the KK towers [12, 14]. It predicts a higher rate for the dimension 6 proton decay than in the usual 4d grand unified theories, which might be seen by further running of the Super-Kamiokande experiment or at a next generation proton decay detector [12]. The soft supersymmetry-breaking masses for the squarks and sleptons are vanishing at the compactification scale, providing a solution to the supersymmetric flavor problem.

3.2 Matter in the bulk

In this sub-section, we introduce matter in the 5d bulk instead of on the $y = 0$ brane. One might naively think that we have only to introduce hypermultiplets $\mathcal{T}_{\mathbf{10}}$ and $\mathcal{F}_{\bar{\mathbf{5}}}$ for each generation, to obtain the correct low energy matter content. However, this does not work because of an automatic "double-triplet splitting" mechanism operating in this setup. Let us, for example, consider $\mathcal{F}_{\bar{\mathbf{5}}} = (F_{\bar{\mathbf{5}}}(+), F^c_{\bar{\mathbf{5}}}(-))$ transforming as $\mathcal{F}_{\bar{\mathbf{5}}} \to \mathcal{F}_{\bar{\mathbf{5}}} P^{-1}$ under $y \to y + 2\pi R$, where the signs in the parentheses represent the parities under $y \to -y$. After the orbifolding, this hypermultiplet gives only one quasi zero-mode, which is the lepton doublet L of the standard model. Thus we do not obtain the correct matter content, D and L. To evade this problem, we can introduce another hypermultiplet $\mathcal{F}'_{\bar{\mathbf{5}}} = (F'_{\bar{\mathbf{5}}}(+), F'^c_{\bar{\mathbf{5}}}(-))$ which transforms as $\mathcal{F}'_{\bar{\mathbf{5}}} \to -\mathcal{F}'_{\bar{\mathbf{5}}} P^{-1}$ under $y \to y + 2\pi R$. This additional hypermultiplet gives D of the standard model as a quasi zero-mode, and completes the standard model matter content. A similar argument applies to the $\mathcal{T}_{\mathbf{10}}$ multiplet: the quasi zero-modes from $\mathcal{T}_{\mathbf{10}}$ are U, E, while that from $\mathcal{T}'_{\mathbf{10}}$ is Q. Therefore, to ensure the correct low energy matter content, we introduce four hypermultiplets, $\mathcal{T}_{\mathbf{10}}, \mathcal{T}'_{\mathbf{10}}, \mathcal{F}_{\bar{\mathbf{5}}}$ and $\mathcal{F}'_{\bar{\mathbf{5}}}$, for each generation [12].

We here explicitly show the boundary conditions for the bulk matter fields in the present model. To do so, we introduce the following notation. We represent $U, E/Q$ components of $\mathcal{T}_{\mathbf{10}}$ (and corresponding states of $\mathcal{T}'_{\mathbf{10}}$) by the superscripts $(+)/(-)$, respectively. We also denote L and D components of $\mathcal{F}_{\bar{\mathbf{5}}}$ (and corresponding states of $\mathcal{F}'_{\bar{\mathbf{5}}}$) by $(+)$ and $(-)$ superscripts. Then, under $y \leftrightarrow -y$, they subject to

$$\begin{pmatrix} T^{(\pm)}_{\mathbf{10_j}} \\ T^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, -y) = \begin{pmatrix} T^{(\pm)}_{\mathbf{10_j}} \\ -T^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y), \tag{39}$$

$$\begin{pmatrix} T'^{(\pm)}_{\mathbf{10_j}} \\ T'^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, -y) = \begin{pmatrix} T'^{(\pm)}_{\mathbf{10_j}} \\ -T'^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y), \tag{40}$$

where $j = 1, 2, 3$ represents the generation index. The boundary condition under $y \leftrightarrow y + 2\pi R$

is given by

$$\begin{pmatrix} \phi^{(\pm)}_{\mathbf{10_j}} \\ \phi^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y + 2\pi R) = \pm\, e^{-2\pi i \alpha \sigma_2} \begin{pmatrix} \phi^{(\pm)}_{\mathbf{10_j}} \\ \phi^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y), \tag{41}$$

$$\begin{pmatrix} \phi'^{(\pm)}_{\mathbf{10_j}} \\ \phi'^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y + 2\pi R) = \mp\, e^{-2\pi i \alpha \sigma_2} \begin{pmatrix} \phi'^{(\pm)}_{\mathbf{10_j}} \\ \phi'^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y), \tag{42}$$

$$\begin{pmatrix} \psi^{(\pm)}_{\mathbf{10_j}} \\ \psi^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y + 2\pi R) = \pm \begin{pmatrix} \psi^{(\pm)}_{\mathbf{10_j}} \\ \psi^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y), \tag{43}$$

$$\begin{pmatrix} \psi'^{(\pm)}_{\mathbf{10_j}} \\ \psi'^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y + 2\pi R) = \mp \begin{pmatrix} \psi'^{(\pm)}_{\mathbf{10_j}} \\ \psi'^{c(\pm)\dagger}_{\mathbf{10_j}} \end{pmatrix}(x^\mu, y), \tag{44}$$

where $\phi^{(\pm)}_{\mathbf{10_j}}$ and $\psi^{(\pm)}_{\mathbf{10_j}}$ ($\psi'^{(\pm)}_{\mathbf{10_j}}$ and $\psi'^{(\pm)}_{\mathbf{10_j}}$) are the scalar and fermion components of $T^{(\pm)}_{\mathbf{10_j}}$ ($T'^{(\pm)}_{\mathbf{10_j}}$), respectively. The same boundary condition also applies to $\mathcal{F}_{\bar{\mathbf{5}}}$ and $\mathcal{F}'_{\bar{\mathbf{5}}}$ fields.

The above boundary conditions precisely give the quarks and leptons in the minimal supersymmetric standard model. Together with the Yukawa couplings

$$\begin{aligned} S = & \int d^4x\, dy\, \delta(y) \Bigg[\int d^2\theta \sum_{j,k=1}^{3} \Big((y_1^1)_{jk} T_{\mathbf{10_j}} T_{\mathbf{10_k}} H_{\mathbf{5_1}} + (y_1^2)_{jk} T_{\mathbf{10_j}} T'_{\mathbf{10_k}} H_{\mathbf{5_1}} + (y_1^3)_{jk} T'_{\mathbf{10_j}} T'_{\mathbf{10_k}} H_{\mathbf{5_1}} \\ & + (y_2^1)_{jk} T_{\mathbf{10_j}} F_{\bar{\mathbf{5}}_\mathbf{k}} H^c_{\mathbf{5_2}} + (y_2^2)_{jk} T_{\mathbf{10_j}} F'_{\bar{\mathbf{5}}_\mathbf{k}} H^c_{\mathbf{5_2}} + (y_2^3)_{jk} T'_{\mathbf{10_j}} F_{\bar{\mathbf{5}}_\mathbf{k}} H^c_{\mathbf{5_2}} + (y_2^4)_{jk} T'_{\mathbf{10_j}} F'_{\bar{\mathbf{5}}_\mathbf{k}} H^c_{\mathbf{5_2}} \Big) + \text{h.c.} \Bigg] \end{aligned} \tag{45}$$

the theory reduces to the minimal supersymmetric standard model at low energies. The soft supersymmetry-breaking parameters (and the μ parameter) at the compactification ($\simeq$ unification) scale are given by Eqs. (28, 29). Here electroweak symmetry breaking can occur naturally in this theory with $\tan\beta \simeq 4$, without the need to make the top squarks very heavy.

We finally discuss the phenomenologies of the model with matters in the bulk. In this case, the quarks and leptons which would be unified into a single multiplet in the usual 4d grand unified theories come from different $SU(5)$ multiplets. Specifically, D and L (Q and U, E) come from different (hyper)multiplets. Therefore, proton decay from broken gauge boson exchange is absent at leading order [12]. Furthermore, there is no unwanted $SU(5)$ relation among the low energy Yukawa couplings arising from the interactions given in Eq. (45) [12]. This is reminiscent of the situation in certain string motivated theories [15]. Nevertheless, the theory still keeps the desired features of the usual 4d grand unified theory: the quantization of hypercharge and the unification of the three gauge couplings [12]. Therefore, this type of theory, with matter in the bulk, preserves (experimentally) desired features of 4d grand unified theories, while not necessarily having the problematic features, such as proton decay and fermion mass relations.

4 Conclusions

In this paper we have introduced a new implementation of the boundary condition supersymmetry breaking mechanism, which allows for a large energy desert in which physics is described by a 4d effective theory with softly broken supersymmetry, such as the minimal supersymmetric standard model. This is accomplished by having a boundary condition which mixes components of superfields in a supersymmetry breaking way by an extremely small angle, α.

In general, we are interested in a higher dimensional supersymmetric field theory which leads to two Higgs doublet zero-modes where there is an orbifold symmetry with the group element

$$U = e^{2\pi i \alpha T}; \qquad T = T_R + r T_H. \tag{46}$$

under $y \to y + 2\pi R$. Here, T_R is a generator which acts non-trivially within a supermultiplet of the higher dimensional theory, and T_H is a generator which mixes up the two Higgs supermultiplets. The parameter r is of order unity, so that the generator T of the orbifold symmetry is a generic linear combination of T_R and T_H. In 5d, there is a unique choice for these generators, and hence a unique result for the form of the supersymmetry breaking and G_H breaking operators that result in the low energy 4d effective theory, as shown in Eq. (11), where $\gamma = r\alpha$. This single orbifold symmetry provides a unified origin for both soft supersymmetry-breaking parameters and the μ parameter. The result of Eq. (11) is rather robust and relies on there being two light Higgs doublets, as required for gauge coupling unification. It does not change if there are other heavy Higgs hypermultiplets which are mixed with the light ones at order α by orbifolding. It is independent of the gauge group and the spacetime structure of matter, as shown explicitly by our models with gauge groups $SU(3) \times SU(2) \times U(1)$ and $SU(5)$, with matter in the bulk or on the brane. The soft supersymmetry breaking interactions of squarks and sleptons do depend on whether matter is on the brane or in the bulk. For the brane case, the squarks and sleptons are massless with $A = -\alpha/R$, Eq. (14), while in the bulk case the squarks and sleptons have degenerate mass-squareds α^2/R^2 and $A = -3\alpha/R$, Eq. (28).

We have shown that this constrained form for the soft operators leads to successful electroweak symmetry breaking only in certain regions of parameter space. For brane matter, $(1/R, \tan\beta)$ lie on a curve between $(2 \times 10^6 \text{ GeV}, 2)$ and $(6 \times 10^7 \text{ GeV}, 20)$; also, a large compactification scale $1/R \gtrsim 10^{14}$ GeV is allowed for low values of $\tan\beta \simeq 2$. For bulk matter, the corresponding regions with successful electroweak symmetry breaking are $(700 \text{ GeV}, 2)$ to $(2 \text{ TeV}, 20)$, and, for larger compactification scales $(10^7 \text{ GeV}, 2)$ to $(10^{16} \text{ GeV}, 4)$. The unified theory prefers the case of bulk matter, since it gives electroweak symmetry breaking with less fine tuning.

Our supersymmetry breaking mechanism solves both the supersymmetric flavor and CP problems — by construction the phases α and γ are real and flavor conserving. If matter is on the brane, the squark and slepton masses arise from renormalization group scaling with flavor blind gauge interactions. With matter in the bulk, the orbifold symmetries necessarily lead to squark

and slepton mass matrices proportional to the unit matrix. Non-trivial flavor mixing boundary conditions are inconsistent. However, flavor and CP violating scalar mass matrices could result in the case of bulk matter with large brane kinetic terms.

With an $SU(5)$ gauge group, the orbifold symmetry can be taken to be

$$U = e^{2\pi i \alpha T} \otimes P \tag{47}$$

where P is the parity $(-,-,-,+,+)$ acting on the **5** of $SU(5)$. This single orbifold symmetry now breaks $SU(5)$ to the standard model gauge group, as well as breaking G_H and supersymmetry. We have explicitly constructed the unique 5d theory which accomplishes this — the only variations being the location of the matter multiplets.

In this viewpoint the hierarchy problem is transformed to the question of the origin of the small non-zero value of the orbifold mixing angle α. The spacetime geometry is not fixed, but is ultimately controlled by certain background fields, and the solution to the hierarchy problem must be sought in the dynamics of these fields.

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